

CHAPTER 2

Surds and Indices

2.1

Radicals

A radical expression (or simply a radical) is an expression of the type $\sqrt[n]{x}$. The sign ' $\sqrt[n]{}$ ' is called the radical sign. The number under this sign, i.e., 'x' is called the radicand and the number in the angular part of the sign, i.e., 'n' is the order of the radical. At present, we shall deal only with cases where x is a real number. Depending on the values of x, n can have certain corresponding values. Initially we shall consider only positive integral values of n. But remember that $\sqrt[n]{x} = x^{1/n}$. We have the following results for radicals.

If $x > 0$, n can have real value except zero.

If $x = 0$, n can have any positive real value.

If $x < 0$, n can have any real value. But we shall consider only rational values.

- $\sqrt{2}$, $\sqrt[3]{3}$, $\sqrt[4]{7}$, $\sqrt[2]{8}$, are some radicals. $\sqrt{0}$, $\sqrt[3]{0}$ are also radicals. $\sqrt[0]{0}$ and $\sqrt[0]{-2}$, $\sqrt[5]{-64}$, $\sqrt[7]{-128}$ are also radicals. $\sqrt[2]{-4}$, $\sqrt[4]{-16}$ are also radicals, but because they are not real numbers, we shall study them only in higher classes. In all these examples, the value of n, i.e., the order of the radical is a positive integer. But as stated above, it can have other values.

2.2

Description of surds

If a is a positive rational number, which is not the nth power (n is any natural number) of any rational number, then the irrational number $\pm \sqrt[n]{a}$ are called simple surds or monomial surds. Every surd is an irrational number (but every irrational number is not a surd). So, the representation of monomial surd on a number line is same that of irrational numbers. e.g.,

- $\sqrt{3}$ is a surd and $\sqrt{3}$ is an irrational number.
- $\sqrt[3]{5}$ is surd and $\sqrt[3]{5}$ is an irrational number.
- π is an irrational number, but it is not a surd.
- $\sqrt[3]{3+\sqrt{2}}$ is an irrational number. It is not a surd, because $3+\sqrt{2}$ is not a rational number.

NUMERICAL CHALLENGE 2.1

Which of the following are surds?

(i) $\sqrt{9}$

(ii) $\sqrt[3]{13}$

(iii) $\sqrt[4]{25}$

(iv) $\sqrt[6]{32}$

Solution

(i) $\sqrt{9} = 3$ is not a surd.

(ii) $\sqrt[3]{13}$ is a surd.

(iii) $\sqrt[4]{25}$ is a surd.

(iv) $\sqrt[6]{32}$ is a surd.

2.3

Types of surds

Unit surds and multiples of surds

If $\sqrt[n]{a}$ is a surd, it is also referred to as a unit surd. If k is a rational number, $k\sqrt[n]{a}$ is a multiple of a surd.

- All multiples of surds can be expressed as unit surds as $k\sqrt[n]{a} = \sqrt[n]{k^n \cdot a}$.

Mixed surds

If a is a rational number (not equal to 0) and $\sqrt[n]{b}$ is a surd, then $a + \sqrt[n]{b}$, $a - \sqrt[n]{b}$ are called mixed surds.

If $a = 0$, they are called pure surds. e.g., $2 + \sqrt{3}$, $5 - \sqrt[3]{6}$ are mixed surds, while $\sqrt{3}$, $\sqrt[3]{6}$ are pure surds.

Compound surds

A surd which is the sum or difference of two or more surds is called a compound surd.

e.g., $\sqrt{2} + \sqrt[3]{3}$, $\sqrt{3} + \sqrt[5]{7} - \sqrt[3]{2}$ and $1 + \sqrt{2} - \sqrt{3}$ are compound surds.

Binomial surds

A compound surd consisting of two surds is called a binomial surd.

e.g., $\sqrt{3} + 3\sqrt{5}$, $\sqrt{6} + 4\sqrt{5}$, $\sqrt{8} - \sqrt[3]{7}$

Similar surds

If two surds are different multiples of the same surd, they are called similar surds. Otherwise they are dissimilar surds.

e.g., $2\sqrt{2}$, $5\sqrt{2}$, $7\sqrt{2}$ are similar surds. $(\sqrt{2} + 3\sqrt{3})$, $(2\sqrt{2} + 6\sqrt{3})$ are similar surds and $(1 + \sqrt{2})$, $(2 + 2\sqrt{2})$ are similar surds. $3\sqrt{3}$ and $6\sqrt{5}$ are dissimilar surds.

- Equality of two mixed surds of the form $a + c\sqrt{b}$ and $d + e\sqrt{b}$.

Two mixed surds $a + c\sqrt{b}$ and $d + e\sqrt{b}$ are equal if and only if their respective rational parts and the irrational parts are equal, i.e., $a = d$ and $c = e$.

NUMERICAL CHALLENGE 2.2

Identify the following types of surds:

(i) $\sqrt{6} + 5\sqrt{3}$

(ii) $\sqrt{15} + \sqrt{8} + \sqrt{11}$

(iii) $\sqrt{5}$

(iv) $5 + \sqrt{7}$

Solution

(i) $\sqrt{6} + 5\sqrt{3}$. It is the sum of two surds.

$\therefore$ It is a compound surd of two surds, i.e., a binomial surd.

(ii) $\sqrt{15} + \sqrt{8} + \sqrt{11}$. It is the sum of the three surds.

$\therefore$ It is a compound surd.

(iii) $\sqrt{5}$. It is a monomial surd or a simple surd.

(iv) $5 + \sqrt{7}$. It is the sum of a rational number and a surd.

$\therefore$ It is a mixed surd.

NUMERICAL CHALLENGE 2.3

Which of the following surds are similar?

(i) $2\sqrt{5}$

(ii) $3\sqrt[3]{5}$

(iii) $4\sqrt{5}$

(iv) $5\sqrt[4]{5}$

Solution

$2\sqrt{5}$ and $4\sqrt{5}$ are multiples of the same surd, $\sqrt{5}$.

$\therefore$ They are similar.

2.4

Laws of radicals

If $a > 0$, $b > 0$ and n is a positive rational number, then

$$(a) (\sqrt[n]{a})(\sqrt[n]{b}) = \sqrt[n]{ab}$$

$$(b) \frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$$

$$(c) \sqrt[m]{\sqrt[n]{a}} = m\sqrt[n]{a} = \sqrt[n]{m\sqrt[n]{a}}$$

Laws of radicals are used to

- convert the multiple of a surd into a unit surd.
- convert certain unit surds into multiples of other unit surds.
- express surds of different orders as surds of the same order.

NUMERICAL CHALLENGE 2.4

Express the following surds in their simplest form as multiples of smaller surds:

$$(i) \sqrt[3]{1458}$$

$$(ii) \sqrt[3]{144}$$

$$(iii) \sqrt[4]{1024}$$

Solution

$$(i) \sqrt[3]{1458} = \sqrt[3]{2(9^3)} = 9\sqrt[3]{2}$$

$$(ii) \sqrt[3]{144} = \sqrt[3]{2^4(3^2)} = \sqrt[3]{2^3(2)(3^2)} = 2\sqrt[3]{18}$$

$$(iii) \sqrt[4]{1024} = \sqrt[4]{2^{10}} = \sqrt[4]{(2^8)(2^2)} = \sqrt[4]{2^8} \cdot \sqrt[4]{2^2} = 4\sqrt{2}$$

Order of surd

In the surd $\sqrt[n]{a}$, n is called the order of the surd. Thus, the order of $\sqrt{2}$, $\sqrt[3]{3}$, $\sqrt[4]{5}$ are 2, 3 and 4 respectively.

- The orders of the radicals $\sqrt[4]{9}$, $\sqrt[6]{16}$, $\sqrt[6]{27}$ are 4, 6 and 6 respectively. We note that $\sqrt[4]{9} = \sqrt[4]{3^2} = 3^{2/4} = \sqrt{3}$ is 2. Thus, the order of a surd is not a property of the surd itself, but of the way in which it is expressed.

Comparison of monomial surds

If two simple surds are of the same order, then they can easily be compared. If $a < b$, $\sqrt[n]{a} < \sqrt[n]{b}$ for all positive integral values of n . e.g., $\sqrt[4]{2} < \sqrt[4]{7}$, $\sqrt[3]{3} < \sqrt[3]{5}$, $\sqrt[5]{10} < \sqrt[5]{13}$ etc.

If two simple surds of different orders have to be compared, they have to be expressed as radicals of the same order.

Thus to compare $\sqrt[4]{6}$ and $\sqrt[3]{5}$, we express both as the radicals of 12 (LCM of 3, 4) order.

$$\sqrt[4]{6} = \sqrt[12]{6^3} \text{ and } \sqrt[3]{5} = \sqrt[12]{5^4}$$

$$\text{as } 6^3 < 5^4, \sqrt[4]{6} < \sqrt[3]{5}$$

NUMERICAL CHALLENGE 2.5

Arrange the following in ascending or descending order of magnitude.

$$\sqrt[4]{6}, \sqrt[3]{7}, \sqrt{5}$$

Solution

$\sqrt[4]{6} = 6^{1/4}$, $\sqrt[3]{7} = 7^{1/3}$, $\sqrt{5} = 5^{1/2}$, LCM of the denominators of the exponents of the three terms, 4, 3 and 2 is 12.

Now express the exponent of each term, as a fraction in which the denominator is 12.

$$6^{1/4} = 6^{3/12} = (6^3)^{1/12} = \sqrt[12]{216}$$

$$7^{1/3} = 7^{4/12} = (7^4)^{1/12} = \sqrt[12]{2401}$$

$$5^{1/2} = 5^{6/12} = (5^6)^{1/12} = \sqrt[12]{15625}$$

$$\text{Now } \sqrt[4]{6} = \sqrt[12]{216}, \sqrt[3]{7} = \sqrt[12]{2401}, \sqrt{5} = \sqrt[12]{15625}$$

Hence, their ascending order is

$$\sqrt[12]{216}, \sqrt[12]{2401}, \sqrt[12]{15625}, \text{ i.e., } \sqrt[4]{6}, \sqrt[3]{7}, \sqrt{5}$$

The descending order of magnitude of the given radicals is $\sqrt{5}, \sqrt[3]{7}, \sqrt[4]{6}$.

2.5**Operations of surds****Addition and subtraction of surds**

Addition and subtraction of similar surds can be done using the distributive law, $a\sqrt{c} \pm b\sqrt{c} = (a \pm b)\sqrt{c}$.

NUMERICAL CHALLENGE 2.6

Simplify the following by combining similar surds.

(i) $2\sqrt{5} + 5\sqrt{5}$

(ii) $3\sqrt{6} + \sqrt{216}$

(iii) $2\sqrt{3} - 5\sqrt{12} + 3\sqrt{48}$

Solution

(i) $2\sqrt{5} + 5\sqrt{5} = (2+5)\sqrt{5} = 7\sqrt{5}$

(ii) $3\sqrt{6} + \sqrt{216} = 3\sqrt{6} + \sqrt{6^2(6)} = 3\sqrt{6} + 6\sqrt{6} = 9\sqrt{6}$

(iii) $2\sqrt{3} - 5\sqrt{12} + 3\sqrt{48}$
 $= 2\sqrt{3} - 5\sqrt{2^2(3)} + 3\sqrt{4^2(3)}$
 $= 2\sqrt{3} - 5(2)\sqrt{3} + 3(4)\sqrt{3}$
 $= (2 - 10 + 12)\sqrt{3} = 4\sqrt{3}$

Multiplication and division of surds

Surds of the same order can be multiplied according to the law, $(\sqrt[n]{x})(\sqrt[n]{y}) = \sqrt[n]{xy}$

- When the surds to be multiplied or divided are not of the same order, they have to be necessarily brought to the same order before the operation is performed.

NUMERICAL CHALLENGE 2.7

Simplify the following

(i) $\sqrt{15} \times \sqrt{35}$

(ii) $2\sqrt{3} \div 3\sqrt{27}$

(iii) Multiply $\sqrt[3]{3}$ by $\sqrt[4]{2}$

(iv) Divide $\sqrt[6]{5}$ by $\sqrt[3]{10}$

Solution

(i) $(\sqrt{15})(\sqrt{35}) = \sqrt{(15)(35)} = \sqrt{(5)(3)(5)(7)} = 5\sqrt{21}$

(ii) $2\sqrt{3} \div 3\sqrt{27} = \frac{2\sqrt{3}}{3\sqrt{27}} = \frac{2\sqrt{3}}{(3)\sqrt{3^2(3)}} = \frac{2\sqrt{3}}{(3)(3)\sqrt{3}} = \frac{2}{9}$

(iii) $\sqrt[3]{3} = 3^{1/3}$ and $\sqrt[4]{2} = 2^{1/4}$

The LCM of 3 and 4 is 12

$\therefore 3^{1/3} = 3^{4/12} = \sqrt[12]{3^4}$, $2^{1/4} = 2^{3/12} = \sqrt[12]{2^3}$

$(\sqrt[3]{3})(\sqrt[4]{2}) = (\sqrt[12]{3^4})(\sqrt[12]{2^3}) = \sqrt[12]{(3^4)(2^3)} = \sqrt[12]{(81)(8)} = \sqrt[12]{648}$

(iv) $\sqrt[6]{5} = 5^{1/6}$ and $\sqrt[3]{10} = 10^{1/3}$

LCM of 3 and 6 is 6

$\sqrt[3]{10} = 10^{1/3} = 10^{2/6} = \sqrt[6]{10^2} = \sqrt[6]{100}$

$\therefore \frac{\sqrt[6]{5}}{\sqrt[6]{100}} = \frac{\sqrt[6]{5}}{\sqrt[6]{100}} = \sqrt[6]{\frac{5}{100}} = \sqrt[6]{\frac{1}{20}}$

2.6

Rationalizing factor (RF)

If the product of two surds is a rational number, then each of the two is a RF of the other.

- RF is not unique.
- If one RF of a surd is known, then the product of this factor and any non-zero rational number is also the RF of the given surd.
- It is convenient to use the simplest of all RF's of the given surd to convert it to a rational number.

e.g., (a) $(3\sqrt{3})(\sqrt{3}) = (3)(3) = 9$, a rational number

$\therefore \sqrt{3}$ is a RF of $3\sqrt{3}$

(b) $(\sqrt{3} + \sqrt{2})(\sqrt{3} - \sqrt{2}) = (\sqrt{3})^2 - (\sqrt{2})^2 = 3 - 2 = 1$, a rational number

$\therefore \sqrt{3} - \sqrt{2}$ is a RF of $\sqrt{3} + \sqrt{2}$ and $\sqrt{3} + \sqrt{2}$ is a RF of $\sqrt{3} - \sqrt{2}$

NUMERICAL CHALLENGE 2.8

Find the simplest RF of

(i) $\sqrt[4]{216}$

(ii) $\sqrt[5]{16}$

Solution

(i) $\sqrt[4]{216} = \sqrt[4]{(2^3)(3^3)} = 2^{3/4} \times 3^{3/4}$

So RF = $2^{1/4} \times 3^{1/4} = (2 \times 3)^{1/4} = 6^{1/4} = \sqrt[4]{6}$

$\therefore \sqrt[4]{6}$ is the simplest RF of $\sqrt[4]{216}$.

(ii) $\sqrt[5]{16} = \sqrt[5]{2^4} = 2^{4/5}$

$\therefore$ RF is $2^{1/5} \Rightarrow (2^{4/5})(2^{1/5}) = 2^{5/5} = 2$

$\therefore \sqrt[5]{2}$ is the simplest RF of $\sqrt[5]{16}$.

- $\sqrt[n]{a}$ is a RF of $\sqrt[n]{a^{n-1}}$ and vice-versa.
- $\sqrt[n]{a^m}$ is a RF of $\sqrt[n]{a^{n-m}}$ and vice-versa.
- $\sqrt{a} + \sqrt{b}$ is a RF of $\sqrt{a} - \sqrt{b}$ and vice-versa.
- $\sqrt[3]{a} + \sqrt[3]{b}$ is a RF of $a^{2/3} - a^{1/3} \cdot b^{1/3} + b^{2/3}$ and vice-versa.
- $\sqrt[3]{a} - \sqrt[3]{b}$ is a RF of $a^{2/3} + a^{1/3} \cdot b^{1/3} + b^{2/3}$ and vice-versa.

NUMERICAL CHALLENGE 2.9

Express the following surds with rational denominators:

(i) $\frac{2}{\sqrt{14}}$

(ii) $\frac{2\sqrt[3]{3}}{\sqrt[3]{25}}$

Solution

(i) $\frac{2}{\sqrt{14}} \times \frac{\sqrt{14}}{\sqrt{14}} = \frac{2\sqrt{14}}{14} = \frac{\sqrt{14}}{7}$

(ii) $\frac{2\sqrt[3]{3}}{\sqrt[3]{25}} = \frac{2\sqrt[3]{3}}{\sqrt[3]{25}} \times \frac{\sqrt[3]{5}}{\sqrt[3]{5}} = \frac{2\sqrt[3]{3 \times 5}}{\sqrt[3]{5^3}} = \frac{2\sqrt[3]{15}}{5}$

NUMERICAL CHALLENGE 2.10

Given that $\sqrt{2} = 1.414$, find the value of $\frac{3}{\sqrt{2}}$ upto three decimal places.

Solution

$$\frac{3}{\sqrt{2}} = \frac{3}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{3\sqrt{2}}{2} = 1.5\sqrt{2} = 1.5(1.414) = 2.121$$

2.7

Rationalization of mixed surds

$(a + \sqrt{b})$ is the rationalizing factor of $a - \sqrt{b}$, where a and b are rational.

NUMERICAL CHALLENGE 2.11

Rationalize the denominator of $\frac{2 + \sqrt{5}}{2 - \sqrt{5}}$.

Solution

$$\begin{aligned} \frac{2 + \sqrt{5}}{2 - \sqrt{5}} &= \left(\frac{2 + \sqrt{5}}{2 - \sqrt{5}} \right) \left(\frac{2 + \sqrt{5}}{2 + \sqrt{5}} \right) \\ &= \frac{(2 + \sqrt{5})^2}{(2)^2 - (\sqrt{5})^2} = \frac{4 + 4\sqrt{5} + 5}{4 - 5} = \frac{9 + 4\sqrt{5}}{-1} = -(9 + 4\sqrt{5}) \end{aligned}$$

NUMERICAL CHALLENGE 2.12

Given $\sqrt{2} = 1.414$, $\sqrt{3} = 1.732$, $\sqrt{5} = 2.236$, $\sqrt{6} = 2.449$, $\sqrt{10} = 3.162$

Find the value of $\frac{\sqrt{2}-1}{\sqrt{3}-\sqrt{5}}$ upto three decimal places.

Solution

We have to rationalize the denominator

The RF of $\sqrt{3}-\sqrt{5}$ is $\sqrt{3}+\sqrt{5}$.

$$\begin{aligned}\therefore \frac{\sqrt{2}-1}{\sqrt{3}-\sqrt{5}} &= \left(\frac{\sqrt{2}-1}{\sqrt{3}-\sqrt{5}} \right) \left(\frac{\sqrt{3}+\sqrt{5}}{\sqrt{3}+\sqrt{5}} \right) \\ &= \frac{\sqrt{6}-\sqrt{3}+\sqrt{10}-\sqrt{5}}{(\sqrt{3})^2-(\sqrt{5})^2} = \frac{\sqrt{10}+\sqrt{6}-\sqrt{3}-\sqrt{5}}{3-5} = \frac{3.162+2.449-1.732-2.236}{-2} = -0.822\end{aligned}$$

Conjugate of the surd of the form $(\sqrt{a}+\sqrt{b})$

When two binomial quadratic surds differ only in sign which connect their terms, they are said to be conjugate.

Two binomial quadratic surds $\sqrt{a}+\sqrt{b}$ and $\sqrt{a}-\sqrt{b}$, are called conjugate surds. The product of conjugate surds is rational.

NUMERICAL CHALLENGE 2.13

Write the conjugate of

(i) $\sqrt{3}+\sqrt{5}$

(ii) $5+3\sqrt{7}$

Solution

(i) $\sqrt{3}-\sqrt{5}$ is the conjugate of $\sqrt{3}+\sqrt{5}$ and $\sqrt{3}-\sqrt{5}$ is also the conjugate of $\sqrt{3}+\sqrt{5}$.

(ii) $5-3\sqrt{7}$ is the conjugate of $5+3\sqrt{7}$.

• The following formulae are helpful in finding the rationalizing factors of mixed quadratic and cubic surds.

(i) $(a+b)(a-b) = a^2 - b^2$

(ii) $(a+b)(a^2-ab+b^2) = a^3 + b^3$

(iii) $(a-b)(a^2+ab+b^2) = a^3 - b^3$

NUMERICAL CHALLENGE 2.14

Find the RF

(i) $2^{1/3} + 2^{-1/3}$

(ii) $5^{1/3} - 5^{-1/3}$

Solution

(i) $2^{1/3} + 2^{-1/3}$

Let $a = 2^{1/3}$ and $b = 2^{-1/3}$

$a^3 = (2^{1/3})^3 = 2$, $b^3 = (2^{-1/3})^3 = 2^{-1} = \frac{1}{2}$

but $a^3 + b^3 = (a+b)(a^2-ab+b^2)$

$\therefore a^2 - ab + b^2 = (2^{1/3})^2 - (2^{1/3} \cdot 2^{-1/3}) + (2^{-1/3})^2 = 2^{2/3} - 1 + 2^{-2/3}$

$\therefore$ RF of $2^{1/3} + 2^{-1/3}$ is $2^{2/3} - 1 + 2^{-2/3}$

(ii) $5^{1/3} - 5^{-1/3}$

$\therefore (a^3 - b^3) = (a-b)(a^2+ab+b^2)$

$\therefore$ RF of $5^{1/3} - 5^{-1/3}$ is $(5^{1/3})^2 + (5^{1/3} \cdot 5^{-1/3}) + (5^{-1/3})^2 = 5^{2/3} + 1 + 5^{-2/3}$

Comparison of compound surds

NUMERICAL CHALLENGE 2.15

Between $\sqrt{7} - \sqrt{3}$ and $\sqrt{11} - \sqrt{7}$, which is greater?

Solution

By rationalizing,

$$\sqrt{7} - \sqrt{3} = \frac{(\sqrt{7} - \sqrt{3})(\sqrt{7} + \sqrt{3})}{\sqrt{7} + \sqrt{3}} = \frac{4}{\sqrt{7} + \sqrt{3}}$$

$$\sqrt{11} - \sqrt{7} = \frac{(\sqrt{11} - \sqrt{7})(\sqrt{11} + \sqrt{7})}{\sqrt{11} + \sqrt{7}} = \frac{4}{\sqrt{11} + \sqrt{7}}$$

The numerator of each of the irrational number is 4.

But $\sqrt{11} + \sqrt{7} > \sqrt{7} + \sqrt{3}$

$$\therefore \frac{4}{\sqrt{7} + \sqrt{3}} > \frac{4}{\sqrt{11} + \sqrt{7}}, \sqrt{7} - \sqrt{3} > \sqrt{11} - \sqrt{7}$$

NUMERICAL CHALLENGE 2.16

Compare the surds $A = \sqrt{8} + \sqrt{7}$ and $B = \sqrt{10} + \sqrt{5}$.

Solution

Since there is a positive sign, square both the surds, we get,

$$A^2 = (\sqrt{8} + \sqrt{7})^2 = 8 + 7 + 2\sqrt{56} = 15 + 2\sqrt{56}$$

$$B^2 = (\sqrt{10} + \sqrt{5})^2 = 10 + 5 + 2\sqrt{50} = 15 + 2\sqrt{50}$$

$$\text{as } 56 > 50, \therefore 15 + 2\sqrt{56} > 15 + 2\sqrt{50} \\ \Rightarrow A > B$$

$$\text{i.e., } \sqrt{8} + \sqrt{7} > \sqrt{10} + \sqrt{5}$$

Rationalizing the numerator

NUMERICAL CHALLENGE 2.17

Rationalize the numerator of $\frac{2 - \sqrt{3+x}}{x-1}$.

Solution

Rationalizing factor of $2 - \sqrt{3+x}$ is $2 + \sqrt{3+x}$

$$\therefore \frac{2 - \sqrt{3+x}}{x-1} = \left(\frac{2 - \sqrt{3+x}}{x-1} \right) \left(\frac{2 + \sqrt{3+x}}{2 + \sqrt{3+x}} \right)$$

$$= \frac{(2)^2 - (\sqrt{3+x})^2}{(x-1)(2 + \sqrt{3+x})} = \frac{4 - (3+x)}{(x-1)(2 + \sqrt{3+x})} = \frac{1-x}{(x-1)(2 + \sqrt{3+x})} = \frac{-1}{2 + \sqrt{3+x}}$$

NUMERICAL CHALLENGE 2.18

Express $E = \frac{1}{\sqrt{5} + \sqrt{3} - \sqrt{8}}$ with a rational denominator.

Solution

The denominator is a trinomial surd, an expression having all the three as surds, we group any two of the three terms, say $\sqrt{5}$ and $\sqrt{3}$.

$$\text{Thus } \sqrt{5} + \sqrt{3} - \sqrt{8} = (\sqrt{5} + \sqrt{3}) - \sqrt{8}$$

Consider the product

$$\begin{aligned} [(\sqrt{5} + \sqrt{3}) - \sqrt{8}] [(\sqrt{5} + \sqrt{3}) + \sqrt{8}] &= (\sqrt{5} + \sqrt{3})^2 - (\sqrt{8})^2 \\ &= 5 + 3 + 2\sqrt{5}\sqrt{3} - 8 = 2\sqrt{15} \end{aligned}$$

$$\therefore \frac{1}{\sqrt{5} + \sqrt{3} - \sqrt{8}} = \frac{\sqrt{5} + \sqrt{3} + \sqrt{8}}{(\sqrt{5} + \sqrt{3} - \sqrt{8})(\sqrt{5} + \sqrt{3} + \sqrt{8})} = \frac{(\sqrt{5} + \sqrt{3} + \sqrt{8})}{2\sqrt{15}}$$

Rationalizing the denominator

$$E = \frac{\sqrt{5} + \sqrt{3} + \sqrt{8}}{2\sqrt{15}} \left(\frac{\sqrt{15}}{\sqrt{15}} \right) = \frac{5\sqrt{3} + 3\sqrt{5} + 2\sqrt{30}}{30}$$

NUMERICAL CHALLENGE 2.19

If both a and b are rational numbers, find the value of a and b in each of the following.

$$(i) \frac{3 + \sqrt{5}}{3 - \sqrt{5}} = a + b\sqrt{5}$$

$$(ii) \frac{3 + 2\sqrt{3}}{5 - 2\sqrt{3}} = a + b\sqrt{3}$$

Solution

(i) $3 + \sqrt{5}$ is the rationalizing factor of $3 - \sqrt{5}$.

$$\begin{aligned} \therefore \frac{3 + \sqrt{5}}{3 - \sqrt{5}} &= \frac{3 + \sqrt{5}}{3 - \sqrt{5}} \times \frac{3 + \sqrt{5}}{3 + \sqrt{5}} = \frac{(3 + \sqrt{5})^2}{(3)^2 - (\sqrt{5})^2} \\ &= \frac{9 + 5 + 6\sqrt{5}}{9 - 5} = \frac{14 + 6\sqrt{5}}{4} = \frac{14}{4} + \frac{6}{4}\sqrt{5} = \frac{7}{2} + \frac{3}{2}\sqrt{5} = a + b\sqrt{5} \end{aligned}$$

$$\therefore a = \frac{7}{2} \text{ and } b = \frac{3}{2}$$

(ii) $5 + 2\sqrt{3}$ is the RF of $5 - 2\sqrt{3}$.

$$\begin{aligned} \therefore \frac{3 + 2\sqrt{3}}{5 - 2\sqrt{3}} &= \frac{3 + 2\sqrt{3}}{5 - 2\sqrt{3}} \times \frac{5 + 2\sqrt{3}}{5 + 2\sqrt{3}} = \frac{15 + 10\sqrt{3} + 12 + 6\sqrt{3}}{(5)^2 - (2\sqrt{3})^2} \\ &= \frac{27 + 16\sqrt{3}}{25 - 12} = \frac{27 + 16\sqrt{3}}{13} = a + b\sqrt{3} \end{aligned}$$

$$\therefore a = \frac{27}{13} \text{ and } b = \frac{16}{13}$$

2.8

Square root of a quadratic surd

Consider the real number $a + \sqrt{b}$, where a and b rational numbers and $\sqrt{b}$ is a surd. Equate the square root of $a + \sqrt{b}$ to $\sqrt{x} + \sqrt{y}$, where x and y are rational numbers, i.e.

$$\sqrt{a + \sqrt{b}} = \sqrt{x} + \sqrt{y}$$

Squaring both sides, $a + \sqrt{b} = x + y + 2\sqrt{xy}$

Equating the rational numbers on the two sides of the above equation, we get $a = x + y$ (1)

and equating the irrational numbers, we get

$$\sqrt{b} = 2\sqrt{xy} \text{ (2)}$$

By solving (1) and (2), we get the values of x and y .

Similarly, $\sqrt{a - \sqrt{b}} = \sqrt{x} - \sqrt{y}$

2.9

Square root of a trinomial quadratic surd

Consider the real number $a + \sqrt{b} + \sqrt{c} + \sqrt{d}$, where a is a rational number and $\sqrt{b}$, $\sqrt{c}$ and $\sqrt{d}$ are surds.

$$\sqrt{a + \sqrt{b} + \sqrt{c} + \sqrt{d}} = \sqrt{x} + \sqrt{y} + \sqrt{z}$$

By squaring both sides, and comparing rational and irrational parts on either sides, we get,

$$x + y + z = a$$

and by solving, we get the values of x , y and z as

$$x = \frac{1}{2}\sqrt{\frac{bd}{c}}, y = \frac{1}{2}\sqrt{\frac{bc}{d}} \text{ and } z = \frac{1}{2}\sqrt{\frac{cd}{b}}$$

NUMERICAL CHALLENGE 2.20

Find the square root of

(i) $7 + 4\sqrt{3}$

(ii) $10 + \sqrt{24} + \sqrt{60} + \sqrt{40}$

Solution

(i) Let $\sqrt{7 + 4\sqrt{3}} = \sqrt{x} + \sqrt{y}$

Squaring both the sides,

$$7 + 4\sqrt{3} = x + y + 2\sqrt{xy} \Rightarrow x + y = 7 \text{ and } 2\sqrt{xy} = 4\sqrt{3} \Rightarrow xy = 12$$

By solving, we get $x = 4$ and $y = 3$

$$\sqrt{x} + \sqrt{y} = \sqrt{4} + \sqrt{3} = 2 + \sqrt{3}$$

(ii) Let the given expression be equal to $a + \sqrt{x} + \sqrt{y} + \sqrt{z}$.

As per the method discussed, $a = 10$, $b = 24$, $c = 60$ and $d = 40$

$$x = \frac{1}{2}\sqrt{\frac{bd}{c}} = \frac{1}{2}\sqrt{\frac{24 \times 40}{60}} = 2$$

$$y = \frac{1}{2}\sqrt{\frac{bc}{d}} = \frac{1}{2}\sqrt{\frac{24 \times 60}{40}} = 3$$

$$z = \frac{1}{2}\sqrt{\frac{cd}{b}} = \frac{1}{2}\sqrt{\frac{60 \times 40}{24}} = 5$$

$$\therefore \sqrt{x} + \sqrt{y} + \sqrt{z} = \sqrt{2} + \sqrt{3} + \sqrt{5}$$

Alternative method

$$\begin{aligned}
 &= \sqrt{10 + \sqrt{24} + \sqrt{60} + \sqrt{40}} \\
 &= \sqrt{10 + 2\sqrt{6} + 2\sqrt{15} + 2\sqrt{10}} \\
 &= \sqrt{(2 + 3 + 5) + 2\sqrt{2(3)} + 2\sqrt{3(5)} + 2\sqrt{2(5)}} \\
 &= \sqrt{(\sqrt{2} + \sqrt{3} + \sqrt{5})^2} \\
 &= \sqrt{2} + \sqrt{3} + \sqrt{5}
 \end{aligned}$$

Other important results

- $\sqrt[5]{a + K\sqrt{b}} = \sqrt{\frac{4b^2 + K}{5}} - b + \sqrt{b}$
- $\sqrt[3]{a + b\sqrt{c}} = \sqrt{\frac{b-c}{3}} + \sqrt{c}$
- $(a + \sqrt{b})^{x^2-K} + (a - \sqrt{b})^{x^2-K} = 2a,$
 $a^2 - b = 1 \Rightarrow x = \pm\sqrt{K \pm 1}$

2.10

Indices

Some useful Formulae

- $a^m \times a^n = a^{m+n}$
- $a^m \div a^n = a^{m-n}$
- $(a^m)^n = (a^n)^m = a^{mn}$
- $\left(\frac{a}{b}\right)^{-\frac{m}{n}} = \left(\frac{b}{a}\right)^{\frac{m}{n}}$
- $a^m \div b^{-n} = a^m \times b^n$
- $(\sqrt[n]{a})^n = a$, where 'n' is a positive integer and 'a' a positive rational number
- $\sqrt[n]{a} \sqrt[n]{b} = \sqrt[n]{ab}$, where 'n' is a positive integer and 'a', 'b' are rational numbers.
- $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$, where 'n' is a positive integer and 'a', 'b' are rational numbers.
- $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a} = \sqrt[n]{\sqrt[m]{a}}$, where 'm', 'n' are positive integers and 'a' is a positive rational number
- $\sqrt[n]{\sqrt[m]{(a^k)^m}} = \sqrt[n]{a^k} = \sqrt[kn]{a^{km}}$, where 'm', 'n' are 'k' are positive integers and 'a' is a positive rational number
- $\sqrt{a} \times \sqrt{a} = a$
- $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$
- $(\sqrt{a} + \sqrt{b})^2 = a + b + 2\sqrt{ab}$
- $(\sqrt{a} - \sqrt{b})^2 = a + b - 2\sqrt{ab}$

$$(xv) a + \sqrt{b} = c + \sqrt{d} \Rightarrow a = c \text{ and } b = d$$

$$(xvi) \frac{1}{\sqrt{a}-\sqrt{b}} = \frac{\sqrt{a}+\sqrt{b}}{(\sqrt{a}-\sqrt{b})(\sqrt{a}+\sqrt{b})} = \frac{\sqrt{a}+\sqrt{b}}{a-b}$$

$$(xvii) \frac{1}{\sqrt{a}+\sqrt{b}} = \frac{\sqrt{a}-\sqrt{b}}{(\sqrt{a}+\sqrt{b})(\sqrt{a}-\sqrt{b})} = \frac{\sqrt{a}-\sqrt{b}}{a-b}$$

$$(xviii) \text{ If } x = n(n+1), \text{ then}$$

$$(a) \sqrt{x - \sqrt{x - \sqrt{x - \dots \infty}}} = n$$

$$(b) \sqrt{x + \sqrt{x + \sqrt{x + \dots \infty}}} = (n+1)$$

NUMERICAL CHALLENGE 2.21

Find the value of x

$$(i) x^{x\sqrt{x}} = (x\sqrt{x})^x \quad (ii) 2^{x-2} = 5^{2-x} \quad (iii) \frac{2^{x-1} \cdot 4^{2x+1}}{8^{x-1}} = 64$$

$$(iv) 2^{x+1} \cdot 5^x = 200 \quad (v) \frac{\left(\frac{1}{9}\right)^{2x-1} \cdot (.0081)^{1/3}}{\sqrt{243}} = \left(\frac{1}{3}\right)^{2x-5} \cdot \sqrt[3]{\frac{27^{x-1}}{10000}}$$

Solution

$$(i) x^{x\sqrt{x}} = \left(x^{\frac{3}{2}}\right)^x \Rightarrow x^{x\sqrt{x}} = x^{\frac{3}{2}x}$$

$$x\sqrt{x} = \frac{3}{2}x \Rightarrow \sqrt{x} = \frac{3}{2} \Rightarrow x = \frac{9}{4}$$

$$(ii) 2^{x-2} = 5^{2-x} \Rightarrow 2^{x-2} = \frac{1}{5^{x-2}}$$

$$\Rightarrow 10^{x-2} = 1 \Rightarrow \frac{10^x}{10^2} = 1$$

$$\Rightarrow 10^x = 10^2 \Rightarrow x = 2$$

$$(iii) \frac{2^{x-1} \cdot 4^{2x+1}}{8^{x-1}} = 64$$

$$\Rightarrow \frac{2^{x-1} \cdot 2^{4x+2}}{2^{3x-3}} = 2^6 \quad (\text{Base has been made equal to 2})$$

$$\Rightarrow 2^{x-1+4x+2-3x+3} = 2^6$$

$$\Rightarrow 2^{x-1+4x+2-3x+3} = 2^6$$

$$\Rightarrow 2^{2x+4} = 2^6$$

$$\Rightarrow 2x+4 = 6$$

$$\Rightarrow x = 1$$

$$(iv) 2^{x+1} \cdot 5^x = 200 \Rightarrow 2 \cdot 2^x \cdot 5^x = 200$$

$$\Rightarrow (2.5)^x = 100 \Rightarrow 10^x = 10^2$$

$$\Rightarrow x = 2$$

(v) The given equation is

$$\frac{(3^{-2})^{2x-1} \cdot (3^4 \cdot 10^{-4})^{1/3}}{3^{5/2}} = \frac{3^{-(2x-5)} \cdot 3^{\frac{3(x-1)}{3}}}{10^{4/3}}$$

$$\Rightarrow \frac{3^{-4x+2+4/3-5/2}}{10^{4/3}} = \frac{3^{-2x+5+x-1}}{10^{4/3}}$$

$$\Rightarrow 3^{-4x+5/6} = 3^{-x+4} \Rightarrow -4x + 5/6 = -x + 4 \Rightarrow x = \frac{-19}{18}$$

NUMERICAL CHALLENGE 2.22

If $3^{x+y} = 81$ and $81^{x-y} = 3$, then find the values of x and y .

Solution

We have, $3^{x+y} = 81 = 3^4$ and $(3^4)^{x-y} = 3^1$

$$\Rightarrow x + y = 4 \quad \dots(1)$$

$$\text{and } 4x - 4y = 1 \quad \dots(2)$$

on adding, we get $8x = 17$

$$\Rightarrow x = \frac{17}{8}$$

Substituting the value of 'x' in (1), we get, $y = \frac{15}{8}$

NUMERICAL CHALLENGE 2.23

Prove that $\frac{1}{1+x^{b-a}+x^{c-a}} + \frac{1}{1+x^{a-b}+x^{c-b}} + \frac{1}{1+x^{b-c}+x^{a-c}} = 1$

Solution

$$\text{LHS} = \frac{1}{1+\frac{x^b}{x^a}+\frac{x^c}{x^a}} + \frac{1}{1+\frac{x^a}{x^b}+\frac{x^c}{x^b}} + \frac{1}{1+\frac{x^b}{x^c}+\frac{x^a}{x^c}} = \frac{x^a}{x^a+x^b+x^c} + \frac{x^b}{x^b+x^a+x^c} + \frac{x^c}{x^c+x^b+x^a} = 1$$

$$= \text{RHS}$$

NUMERICAL CHALLENGE 2.24

If $\frac{2^x}{1+2^x} = \frac{1}{4}$, then value of $\frac{8^x}{1+8^x}$.

Solution

$$\frac{2^x}{1+2^x} = \frac{1}{4} \Rightarrow 4 \cdot 2^x = 1 + 2^x$$

$$\Rightarrow 3 \cdot 2^x = 1 \Rightarrow 2^x = \frac{1}{3}$$

$$\therefore \frac{8^x}{1+8^x} = \frac{(2^x)^3}{1+(2^x)^3} = \frac{\frac{1}{27}}{1+\frac{1}{27}} = \frac{1}{28}$$

NUMERICAL CHALLENGE 2.25

If $a^{1/x} = b^{1/y} = c^{1/z}$, $b^2 = ac$, then find the value of $\frac{x+z}{2y}$.

Solution

$$a^{1/x} = k \quad b^{1/y} = k \quad c^{1/z} = k$$

$$a = k^x \quad b = k^y \quad c = k^z$$

we have, $b^2 = ac$

$$(k^y)^2 = k^x \times k^z$$

$$k^{2y} = k^x \times k^z$$

$$\Rightarrow 2y = x + z$$

$$\Rightarrow \frac{x+z}{2y} = 1$$

NUMERICAL CHALLENGE 2.26

Simplify :

$$(i) \frac{(xyz)^4}{(x^{-2}y^3)^{-3} \left(\frac{1}{z^2} \right)^6} = (x \neq 0, y \neq 0, z \neq 0)$$

$$(ii) \left(\frac{x^a}{x^b} \right)^{a+b} \div \left(\frac{x^a}{x^{a-b}} \right)^{\frac{a^2}{b}}$$

$$(iii) (x^{2n-1} + y^{2n-1})(x^{2n-1} - y^{2n-1})$$

Solution

$$(i) \frac{(xyz)^4}{(x^{-2}y^3)^{-3} \left(\frac{1}{z^2} \right)^6} = \frac{x^4 \cdot y^4 \cdot z^4}{x^6 \cdot y^{-9} \cdot z^3} = \frac{y^{9+4} \cdot z^{4-3}}{x^{6-4}} = \frac{y^{13} \cdot z}{x^2}$$

$$(ii) \left(\frac{x^a}{x^b} \right)^{a+b} \div \left(\frac{x^a}{x^{a-b}} \right)^{\frac{a^2}{b}} = x^{(a-b)(a+b)} \div (x^{a-a+b})^{\frac{a^2}{b}}$$

$$= x^{a^2-b^2} \div (x^b)^{\frac{a^2}{b}} = x^{a^2-b^2} \div x^{a^2} = x^{(a^2-b^2-a^2)} = x^{-b^2}$$

(iii) In the given expression, we assume that

$$x^{2^{n-1}} = a \text{ and } y^{2^{n-1}} = b$$

Then it becomes

$$(a + b)(a - b) = a^2 - b^2$$

$$= \left[x^{2^{n-1}} \right]^2 - \left[y^{2^{n-1}} \right]^2$$

$$= x^{2^{n-1} \times 2^1} = a \text{ and } y^{2^{n-1} \times 2^1}$$

$$= x^{2^{n-1+1}} - y^{2^{n-1+1}} = x^{2^n} - y^{2^n}$$

NUMERICAL CHALLENGE 2.27

If $a^x = m$, $a^y = n$ and $a^2 = (m^y n^x)^z$, then find the value of xyz .

Solution

Given that $a^2 = (m^y n^x)^z$

$$\Rightarrow a^2 = [(a^x)^y \cdot (a^y)^x]^z \quad [\because m = a^x \text{ and } n = a^y]$$

$$\Rightarrow a^2 = [a^{xy} \cdot a^{xy}]^z$$

$$\Rightarrow a^2 = (a^{2xy})^z$$

$$\Rightarrow a^2 = a^{2xyz}$$

Since base is same.

$$\text{Hence } 2 = 2xyz$$

$$\Rightarrow xyz = 1$$

NUMERICAL CHALLENGE 2.28

Which is the greater of the two 2^{300} or 3^{200} ?

Solution

For comparing two numbers of different base and different exponents, it is better that either the base or the exponent is made same for both numbers.

$$\text{Hence } 2^{300} = (2^3)^{100} = 8^{100} \quad (\text{exponent is } 100)$$

$$\text{and } 3^{200} = (3^2)^{100} = 9^{100} \quad (\text{exponent is } 100)$$

From the above it is evident that

$$9^{100} > 8^{100}$$

$$\Rightarrow 3^{200} > 2^{300}$$

NUMERICAL CHALLENGE 2.29

Given that $\sqrt[3]{3^x} = 5^{1/4}$ and $\sqrt[4]{5^y} = \sqrt{3}$, then find the value of $2xy$.

Solution

$$\sqrt[3]{3^x} = 5^{1/4}$$

$$\Rightarrow 3^{x/3} = 5^{1/4} \quad \dots(i)$$

$$\text{and } \sqrt[4]{5^y} = \sqrt{3}$$

$$\Rightarrow 5^{y/4} = 3^{1/2}$$

$$\Rightarrow 5 = 3^{2 \times \frac{4}{y}} \quad \dots(ii)$$

Putting the value of 5 from equation (ii) in equation (i)

$$3^{x/3} = \left(3^{2 \times \frac{4}{y}} \right)^{1/4}$$

$$\Rightarrow \frac{x}{3} = \frac{1}{2} \times \frac{4}{y} \times \frac{1}{4}$$

$$\Rightarrow 2xy = 3$$

EXERCISE

Multiple choice questions

1. If $\sqrt{19-4\sqrt{x}} = \sqrt{12}-\sqrt{7}$, then $x =$
 (1) 84 (2) 28 (3) 21 (4) 14
2. $\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\dots\infty}}}} =$
 (1) 3 (2) 2 (3) 1 (4) ± 3
3. If $x = 8 - \sqrt{60}$, then $\frac{1}{2}\left[\sqrt{x} + \frac{2}{\sqrt{x}}\right] =$
 (1) $\sqrt{5}$ (2) $\sqrt{3}$ (3) $2\sqrt{5}$ (4) $2\sqrt{3}$
4. $\sqrt{\sqrt{50}+\sqrt{48}} = k(\sqrt{3}+\sqrt{2})$, then $k =$
 (1) $2^{1/2}$ (2) 2 (3) $2^{1/4}$ (4) None
5. If $\frac{4+3\sqrt{5}}{4-3\sqrt{5}} = a+b\sqrt{5}$, then $(a, b) =$
 (1) $\left(\frac{61}{29}, \frac{-24}{29}\right)$ (2) $\left(\frac{-61}{29}, \frac{24}{29}\right)$ (3) $\left(\frac{61}{29}, \frac{24}{29}\right)$ (4) $\left(\frac{-61}{29}, \frac{-24}{29}\right)$
6. If $x = \sqrt{5}$, $y = \sqrt[4]{10}$, $z = \sqrt[3]{6}$, then
 (1) $x < y < z$ (2) $y < z < x$ (3) $z < x < y$ (4) None
7. If $x = \sqrt[3]{9}$, $y = \sqrt[4]{11}$, $z = \sqrt[5]{17}$ then
 (1) $x > y > z$ (2) $y > z > x$ (3) $z > y > x$ (4) $x < z < y$
8. The smallest of $\sqrt[3]{4}$, $\sqrt[4]{5}$, $\sqrt[4]{6}$, $\sqrt[3]{8}$ is
 (1) $\sqrt[3]{8}$ (2) $\sqrt[4]{5}$ (3) $\sqrt[3]{4}$ (4) $\sqrt[4]{6}$
9. If $\frac{\ell}{\sqrt{10}+\sqrt{14}+\sqrt{15}+\sqrt{21}} = \frac{\sqrt{10}-\sqrt{14}-\sqrt{15}+\sqrt{21}}{k}$, then
 (1) $k = \ell/2$ (2) $\ell = k/2$ (3) $\ell = 2/k$ (4) None
10. The positive square root of $11+\sqrt{112}$ is
 (1) $\sqrt{7}+2$ (2) $\sqrt{7}+\sqrt{2}$ (3) $2-\sqrt{7}$ (4) None
11. If $x = 2 + 2^{1/3} + 2^{2/3}$, then $x^3 - 6x^2 + 6x =$
 (1) 2 (2) 1 (3) 4 (4) 3
12. The square root $5+2\sqrt{6}$ is
 (1) $\sqrt{3}+2$ (2) $\sqrt{3}-\sqrt{2}$ (3) $\sqrt{2}-\sqrt{3}$ (4) $\sqrt{3}+\sqrt{2}$
13. $\sqrt{21-4\sqrt{5}+8\sqrt{3}-4\sqrt{15}} =$
 (1) $\sqrt{5}-2+2\sqrt{3}$ (2) $\sqrt{5}-\sqrt{4}-\sqrt{12}$ (3) $-\sqrt{5}+\sqrt{4}+\sqrt{12}$ (4) $-\sqrt{5}-\sqrt{4}+\sqrt{12}$
14. $\frac{1}{\sqrt{12}+\sqrt{140}} + \frac{1}{\sqrt{8}-\sqrt{60}} =$
 (1) $\frac{2}{\sqrt{10}+\sqrt{84}}$ (2) 0 (3) $\frac{1}{\sqrt{10}+\sqrt{84}}$ (4) $\frac{2}{\sqrt{10}-\sqrt{84}}$

15. If $\sqrt{23+x\sqrt{10}} = \sqrt{18} + \sqrt{5}$ then $x =$
 (1) 6 (2) -6 (3) 2 (4) -2
16. If $\frac{4+3\sqrt{3}}{\sqrt{7+4\sqrt{3}}} = a + \sqrt{b}$, then $(a, b) =$
 (1) (12, -1) (2) (1, 12) (3) (-1, 12) (4) (-12, 1)
17. If $x = 3 - \sqrt{5}$, then $\frac{\sqrt{x}}{\sqrt{2+\sqrt{3x-2}}} =$
 (1) $1/\sqrt{5}$ (2) $\sqrt{5}$ (3) $\sqrt{3}$ (4) $1/\sqrt{3}$
18. Rationalise the denominator of $\frac{a+\sqrt{a^2-1}}{a-\sqrt{a^2-1}}$ is
 (1) $2a^2+2a\sqrt{a^2-1}-1$ (2) $2a\sqrt{a^2-1}-1$ (3) $\sqrt{a^2-1}-1$ (4) $2a-1$
19. $\frac{1}{2-\sqrt{3}} - \frac{1}{\sqrt{3}+\sqrt{2}} + \frac{4}{3-\sqrt{5}} =$
 (1) $3+\sqrt{2}+\sqrt{7}$ (2) $5+\sqrt{2}+\sqrt{5}$ (3) $\sqrt{2}+\sqrt{5}$ (4) $1-\sqrt{5}$
20. If $\frac{4}{2+\sqrt{3}+\sqrt{7}} = \sqrt{a}+\sqrt{b}-\sqrt{c}$, then
 (1) $a = 1, b = 4/3, c = 7/3$ (2) $a = 1, b = 2/3, c = 7/9$
 (3) $a = 2/3, b = 1, c = 7/3$ (4) $a = 7/9, b = 4/3, c = 1$
21. If $x = 5 + 2\sqrt{6}$, then $\sqrt{\frac{x}{2}} - \frac{1}{\sqrt{2x}} =$
 (1) 1 (2) 2 (3) 3 (4) 4
22. $\frac{3\sqrt{2}}{\sqrt{6}-\sqrt{3}} - \frac{4\sqrt{3}}{\sqrt{6}-\sqrt{2}} + \frac{2\sqrt{3}}{\sqrt{6}+2} =$
 (1) 2 (2) 1 (3) 0 (4) -1
23. $\frac{3+\sqrt{6}}{\sqrt{75}-\sqrt{48}-\sqrt{32}+\sqrt{50}} =$
 (1) $\sqrt{2}$ (2) $\sqrt{3}$ (3) $\sqrt{3}+\sqrt{2}$ (4) $\sqrt{3}-\sqrt{2}$
24. If $\sqrt{\frac{6+2\sqrt{3}}{33-19\sqrt{3}}} = a+b\sqrt{3}$, then $a+b =$
 (1) 6 (2) 8 (3) 10 (4) 12
25. If $\frac{\left(p+\frac{1}{q}\right)^p \left(p-\frac{1}{q}\right)^q}{\left(q+\frac{1}{p}\right)^p \left(q-\frac{1}{p}\right)^q} = \left(\frac{p}{q}\right)^x$ then $x =$
 (1) $p-q$ (2) $p+q$ (3) $q-p$ (4) pq

26. $\frac{5+\sqrt{3}}{2-\sqrt{3}} =$
 (1) $3+7\sqrt{3}$ (2) $13+\sqrt{3}$ (3) $1+\sqrt{3}$ (4) $13+7\sqrt{3}$
27. If $\sqrt{10+2\sqrt{21}} = \sqrt{7} + \sqrt{x}$ then $x =$
 (1) 4 (2) 3 (3) 1 (4) 0
28. $x = \sqrt{\frac{7+4\sqrt{3}}{7-4\sqrt{3}}}$, then $x + \frac{1}{x} =$
 (1) 0 (2) -1 (3) 14 (4) 2
29. If $3 + \sqrt{27} + \sqrt{75} = a + b\sqrt{3}$ then $(a, b) =$
 (1) 3, 4 (2) 3, 8 (3) 8, 1 (4) 4, 2
30. If $\sqrt[3]{6y-9} = 3$ then $y =$
 (1) 1 (2) 2 (3) 6 (4) 0
31. If $a = \frac{1}{3-2\sqrt{2}}$, $b = \frac{1}{3+2\sqrt{2}}$ then $a^2b + ab^2 =$
 (1) 0 (2) 6 (3) -6 (4) 1
32. $x = 7 + 4\sqrt{3}$ and $xy = 1$ then $\frac{1}{x^2} + \frac{1}{y^2} =$
 (1) 194 (2) 19 (3) 192 (4) 14
33. If $\sqrt[3]{a} + \sqrt[3]{b} + \sqrt[3]{c} = 0$, then $(a + b + c)^3 =$
 (1) abc (2) $3abc$ (3) $9ac$ (4) $27abc$
34. If $x = 5 + 2\sqrt{6}$ then $\frac{x-1}{\sqrt{x}} =$
 (1) $\sqrt{2}$ (2) $2\sqrt{2}$ (3) $\sqrt{3}$ (4) $2\sqrt{3}$
35. The value of the expression: $\frac{(x^{a+b})^2 \times (x^{b+c})^2 \times (x^{c+a})^2}{(x^a x^b x^c)^4}$, if $x = 2$, $a = 1$, $b = 2$, $c = 3$ is
 (1) 1 (2) 8 (3) 0 (4) None of these
36. If $4^x = 5^y = 20^z$, then the value of z is
 (1) $\frac{2xy}{x+y}$ (2) $-2xy$ (3) $2xy$ (4) $\frac{xy}{x+y}$
37. If $x^a \cdot x^b \cdot x^{3ab(a+b)} = (2^5)^{25}$ and $a + b = 5$, then the value of x is
 (1) 2 (2) 0 (3) -2 (4) -1
38. The numerical value of $\left(x^{\frac{1}{a-b}}\right)^{\frac{1}{c-a}} \times \left(x^{\frac{1}{b-c}}\right)^{\frac{1}{a-b}} \times \left(x^{\frac{1}{c-a}}\right)^{\frac{1}{b-c}}$ is
 (1) 1 (2) 8 (3) 0 (4) None of these
39. The value of $\frac{(x^{2n-1} + y^{2n-1})(x^{2n-1} - y^{2n-1})}{x^{2n} - y^{2n}}$ is
 (1) 1 (2) 8 (3) 0 (4) None of these
40. $\left(\left((625)^{-\frac{1}{2}}\right)^{-\frac{1}{4}}\right)^2 =$
 (1) 4 (2) 5 (3) 2 (4) 3

41. $\left(5\left(8^{\frac{1}{3}}+27^{\frac{1}{3}}\right)^3\right)^{\frac{1}{4}} =$
 (1) 3 (2) 6 (3) 5 (4) 4
42. $(0.000729)^{-3/4} \times (0.09)^{-3/4} =$
 (1) $\frac{10^3}{3^3}$ (2) $\frac{10^5}{3^5}$ (3) $\frac{10^2}{3^2}$ (4) $\frac{10^6}{3^6}$
43. If $800 = 8 \times 10^8 \times x^{-3/2}$, then $x =$
 (1) 10^2 (2) 10^3 (3) 10^4 (4) 10^5
44. $\left(-\frac{5}{6}\right)^{3/4}$ when divided by $\left(-\frac{5}{6}\right)^{7/6}$ becomes $\left(-\frac{5}{6}\right)^{7-x}$, the value of x is
 (1) $\frac{7}{12}$ (2) $\frac{89}{12}$ (3) $\frac{8}{12}$ (4) $\frac{10}{12}$
45. If $4^x - 4^{x-1} = 24$, then the value of $(2x)^x$ is
 (1) $5^{5/2}$ (2) $5^{1/3}$ (3) $4^{1/2}$ (4) $6^{1/2}$
46. $\left(7^{-\frac{1}{2}} \times 5^2\right)^2 \div \sqrt{25^3} =$
 (1) $\frac{5}{7}$ (2) $\frac{7}{5}$ (3) 35 (4) $-\frac{5}{7}$
47. $(2d^2e^{-1})^3 \times \left(\frac{d^3}{e}\right)^{-2} =$
 (1) $8e^{-2}$ (2) $8e^{-3}$ (3) $8e^{-1}$ (4) $8e^{-4}$
48. If $9^{1/2} \times 3^m = (27)^m$, then the value of m is
 (1) $1/4$ (2) $1/5$ (3) $1/3$ (4) $1/2$
49. If $\sqrt[3]{9^x} = \sqrt[3]{9^2}$, then $x =$
 (1) $\frac{2}{3}$ (2) $\frac{4}{3}$ (3) $\frac{1}{3}$ (4) $\frac{5}{3}$
50. If $a = x + \frac{1}{x}$, then $x^3 + x^{-3} =$
 (1) $a^3 + 3a$ (2) $a^3 - 3a$ (3) $a^3 + 3$ (4) $a^3 - 3$
51. If $a = x^{1/3} - x^{-1/3}$, then $a^3 + 3a =$
 (1) $x - x^{-1}$ (2) $x^{-1} - x$ (3) $x + x^{-1}$ (4) $x^{-1} + x$
52. If $a + b + c = 0$, then $(a^3 + b^3 + c^3)^2 =$
 (1) $3a^2b^2c^2$ (2) $9abc$ (3) $9a^2b^2c^2$ (4) $27abc$
53. If $a^x = b^y = c^z$, $abc = 1$, then the value of $xy + yz + zx$ is
 (1) 1 (2) 2 (3) 0 (4) None of these
54. If $a^{x-1} = bc$, $b^{y-1} = ac$, $c^{z-1} = ab$, then the value of $xy + yz + zx - xyz$ is
 (1) 1 (2) 2 (3) 0 (4) None of these
55. If $(a^m)^n = a^{m^n}$, then express 'm' in the terms of n is
 (1) $n^{\left(\frac{1}{n-1}\right)}$ (2) $n^{\left(\frac{1}{n+1}\right)}$ (3) $n^{\left(\frac{1}{n}\right)}$ (4) None of these

56. If $\left(\frac{q}{p}\right)^{1-2x} = \sqrt{\frac{p}{q}}$ then the value x is
 (1) $3/4$ (2) $4/3$ (3) 1 (4) None of these
57. The value of $\left\{\frac{a^{q-p}}{\sqrt[q]{a^{pq-q^2}}} \times a^{2(p-q)}\right\}^n$ is
 (1) 2 (2) p (3) 1 (4) None of these
58. If $a + b + c = 0$, then $x^{a^2b^{-1}c^{-1}} \cdot x^{a^{-1}b^2c^{-1}} \cdot x^{a^{-1}b^{-1}c^2} =$
 (1) $x^{a^2b^2c^2}$ (2) $x^{1/a^2b^2c^2}$ (3) $x^{1/2}$ (4) x^3
59. If $a^x = b^y = c^z = abc$, then $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} =$
 (1) -1 (2) xyz (3) 1 (4) $\frac{1}{xyz}$
60. $2^3 4^5 =$
 (1) 2^{60} (2) 2^{932} (3) $(2^3)^{20}$ (4) None of these
61. $\frac{1}{1+x^{a-b}} + \frac{1}{1+x^{b-a}} =$
 (1) $\frac{x^{ab}}{x^a + x^b}$ (2) $\frac{x^{ab}}{x^{a-b}}$ (3) 1 (4) $\frac{x^{ab}}{x^{b-a}}$
62. $\frac{2 \cdot 3^{n+1} + 7 \cdot 3^{n-1}}{3^{n+2} - 2\left(\frac{1}{3}\right)^{1-n}} =$
 (1) 1 (2) 3 (3) -1 (4) 0
63. If $\frac{(2^{n+1})^m (2^2)^n 2^n}{(2^{m+1})^n 2^{2m}} = 1$, then m =
 (1) 0 (2) 1 (3) n (4) 2n
64. If $abc = 1$, then $(1 + a + b^{-1})^{-1} + (1 + b + c^{-1})^{-1} + (1 + c + a^{-1})^{-1} =$
 (1) 0 (2) 1 (3) -1 (4) $a + b + c$
65. If $x^y = y^x$, then $\left(\frac{x}{y}\right)^{x/y} =$
 (1) 0 (2) 1 (3) $\frac{x}{y} - 1$ (4) $\frac{x}{y}$
66. The value of $\left(\frac{x^a}{x^b}\right)^{a^2+ab+b^2} \left(\frac{x^b}{x^c}\right)^{b^2+bc+c^2} \left(\frac{x^c}{x^a}\right)^{c^2+ac+a^2}$ is
 (1) 1 (2) 2 (3) 3 (4) All of these
67. The value of $\frac{3 \cdot 2^n - 4 \cdot 2^n}{2^n - 2^{n-1}}$ is
 (1) -1 (2) -3 (3) -4 (4) -2

68. The value of $\frac{p+p^2+p^3+p^4+p^5+p^6+p^7}{p^{-3}+p^{-4}+p^{-5}+p^{-6}+p^{-7}+p^{-8}+p^{-9}}$ is
 (1) p^{10} (2) 1 (3) 0 (4) p^3
69. $\left(\frac{x^a}{x^b}\right)^{a+b} \times \left(\frac{x^b}{x^c}\right)^{b+c} \times \left(\frac{x^c}{x^a}\right)^{c+a} =$
 (1) -1 (2) 1 (3) 2 (4) -2
70. $\left(\frac{64}{125}\right)^{-2/3} \div \frac{1}{(256/625)^{1/4}} + \left(\frac{\sqrt{25}}{\sqrt[3]{64}}\right)^0 =$
 (1) $\frac{9}{2}$ (2) $\frac{9}{4}$ (3) $\frac{8}{2}$ (4) $\frac{8}{4}$
71. If $\frac{4^{n+1} \cdot 2^n - 8^n}{2^{2n}} = \frac{3}{8}$, then $n =$
 (1) -1 (2) -2 (3) -3 (4) -4
72. The simplest value of $\frac{\left(p^2 - \frac{1}{q^2}\right)^p \left(p - \frac{1}{q}\right)^{q-p}}{\left(q^2 - \frac{1}{p^2}\right)^q \left(q + \frac{1}{p}\right)^{p-q}}$ is
 (1) 1 (2) $\left(\frac{p}{q}\right)^{p-q}$ (3) $\left(\frac{p}{q}\right)^{p+q}$ (4) -1
73. The value of $\frac{1}{1+x^{b-c}+x^{a-c}} + \frac{1}{1+x^{c-a}+x^{b-a}} + \frac{1}{1+x^{a-b}+x^{c-b}}$ is
 (1) 1 (2) 8 (3) 0 (4) None of these
74. The value of $\left(\frac{x^m}{x^n}\right)^{m^2+mn+n^2} \times \left(\frac{x^n}{x^\ell}\right)^{n^2+n\ell+\ell^2} \times \left(\frac{x^\ell}{x^m}\right)^{\ell^2+\ell m+m^2}$ is
 (1) 1 (2) 8 (3) 0 (4) None of these
75. If $pqr = 1$, then the value of $\frac{1}{1+p+q^{-1}} + \frac{1}{1+q+r^{-1}} + \frac{1}{1+r+p^{-1}}$ is
 (1) 1 (2) 8 (3) 0 (4) None of these

ANSWERS

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	3	1	1	3	4	2	1	2	3	1	1	4	3	4	1
Que.	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Ans.	3	1	1	2	1	2	3	2	2	2	4	2	3	2	3
Que.	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
Ans.	2	1	4	2	1	4	1	1	1	2	3	4	3	2	1
Que.	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
Ans.	1	3	4	2	2	1	3	3	3	1	1	3	4	3	4
Que.	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75
Ans.	3	1	4	2	3	1	4	1	2	2	3	3	1	1	1